Name ANSWERS	Class

MATHS TEACHER HUB

www.MathsTeacherHub.com

Circles

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

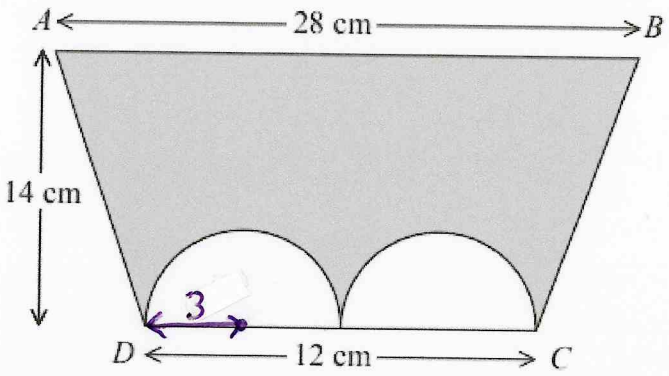
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

1 The diagram shows a trapezium $ABCD$ and two identical semicircles.



The centre of each semicircle is on DC.

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

Trapezium = $\frac{1}{2}(12+28) \times 14 = 280\text{cm}^2$

Two semicircles make a whole circle.

Circle area = $\pi \times 3^2 = 9\pi$

Area of shaded region = $280 - 9\pi$

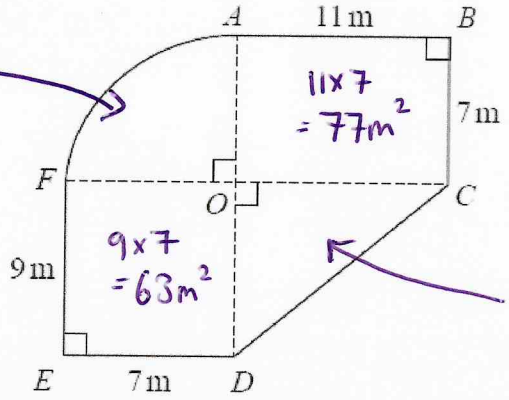
$= 251.7256661\text{ cm}^2$

252 cm²

(Total for Question 1 is 4 marks)

4 The diagram shows a plan of Jason's garden.

ABCO and *DEFO* are rectangles.
CDO is a right-angled triangle.
AFO is a sector of a circle with centre *O* and angle *AOF* = 90°



$\frac{1}{4} \times (\pi \times 7^2)$
$= 38.4845 m^2$

11×7
$= 77 m^2$

9×7
$= 63 m^2$

$\frac{11 \times 7}{2} = 49.5 m^2$

Jason is going to cover his garden with grass seed.
Each bag of grass seed covers 14 m² of garden.
Each bag of grass seed costs £10.95

Work out how much it will cost Jason to buy all the bags of grass seed he needs.

Total area = $38.4845 + 77 + 63 + 49.5$

$= 227.98451 m^2$

Number of bags = $\frac{227.98451}{14} = 16.2846$ bags

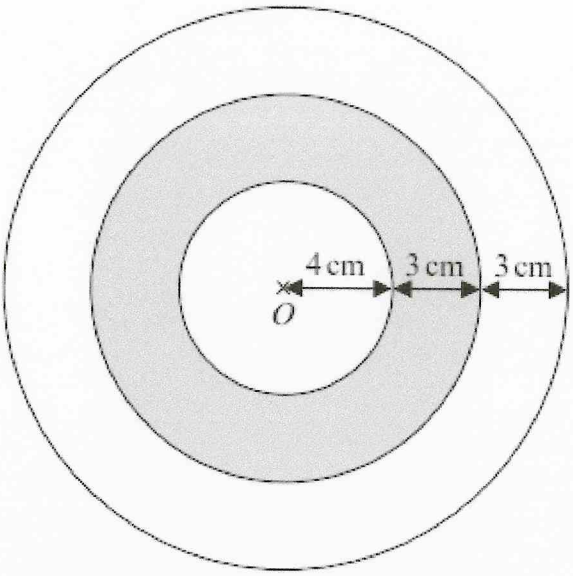
17 bags needed

$17 \times £10.95 = £186.15$

£ 186.15

(Total for Question 4 is 5 marks)

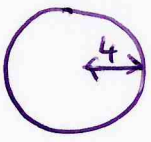
4 The diagram shows a logo made from three circles.



Each circle has centre O.

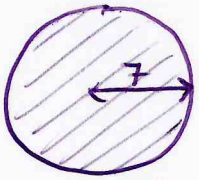
Daisy says that exactly $\frac{1}{3}$ of the logo is shaded.

Is Daisy correct?
You must show all your working.



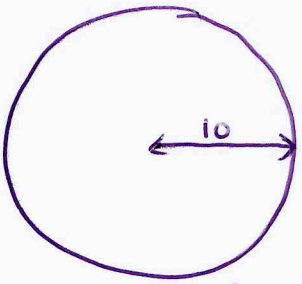
$\pi \times 4^2$

$= 16\pi$



$\pi \times 7^2$

$= 49\pi$



$\pi \times 10^2$

$= 100\pi$

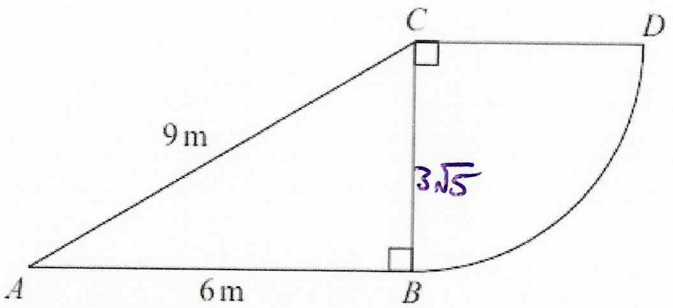
Shaded area $= 49\pi - 16\pi = 33\pi$

- $\frac{33\pi}{100\pi} = \frac{33}{100} = 33\%$
- $\frac{1}{3} = 33.\dot{3}\%$

Daisy is incorrect.

(Total for Question 4 is 4 marks)

7 The diagram shows a right-angled triangle and a quarter circle.



The right-angled triangle ABC has angle $ABC = 90°$
The quarter circle has centre C and radius CB.

Work out the area of the quarter circle.
Give your answer correct to 3 significant figures.
You must show all your working.

$CB = \sqrt{9^2 - 6^2}$

$= 3\sqrt{5}$

Area of the quarter circle $= \frac{1}{4} \times (\pi \times r^2)$

$= \frac{1}{4} \times (\pi \times (3\sqrt{5})^2)$

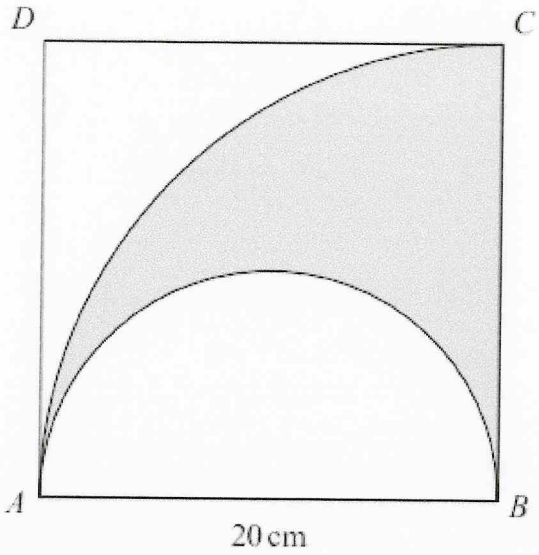
$= \frac{45}{4}\pi$

$= 35.34291735 \text{ m}^2$

35.3 .... m^2

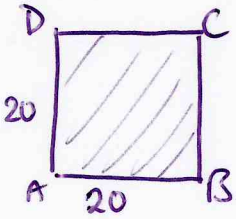
(Total for Question 7 is 4 marks)

7 The diagram shows a square $ABCD$ with sides of length 20 cm.
It also shows a semicircle and an arc of a circle.

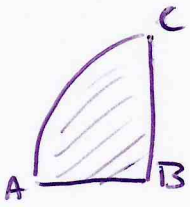


AB is the diameter of the semicircle.
AC is an arc of a circle with centre B.

Show that $\dfrac{\text{area of shaded region}}{\text{area of square}} = \dfrac{\pi}{8}$

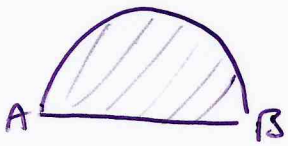


$20 \times 20 = 400\text{cm}^2$



$\frac{1}{4}(\pi \times 20^2)$

$= 100\pi$



$\frac{1}{2}(\pi \times 10^2)$

$= 50\pi$

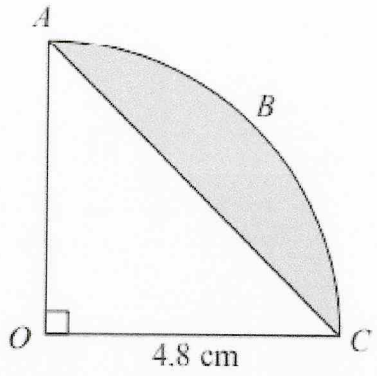
Shaded region $= 100\pi - 50\pi$

$= 50\pi$

$\therefore \dfrac{50\pi}{400} = \dfrac{5\pi}{40} = \dfrac{\pi}{8}$

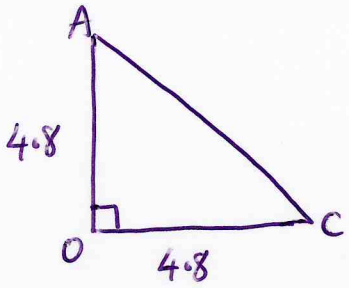
(Total for Question 7 is 4 marks)

7



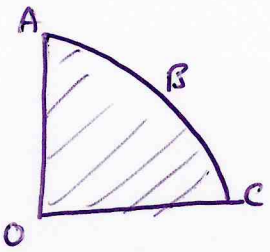
The arc ABC is a quarter of a circle with centre O and radius 4.8 cm.
AC is a chord of the circle.

Work out the area of the shaded segment.
Give your answer correct to 3 significant figures.



$$\frac{4.8 \times 4.8}{2}$$

$= 11.52\text{cm}^2$



$$\frac{1}{4}(\pi \times 4.8^2)$$

$= 18.09557368\text{cm}^2$

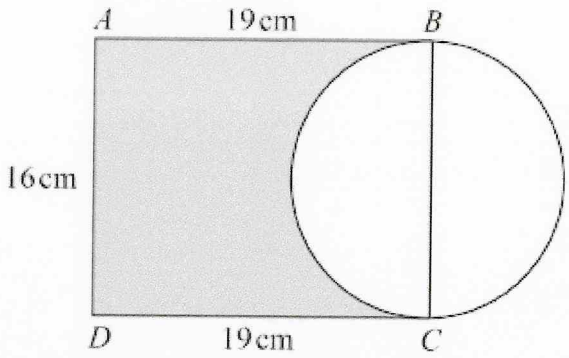
Shaded segment $= 18.09557 - 11.52$

$= 6.57557368$

6.58 cm^2

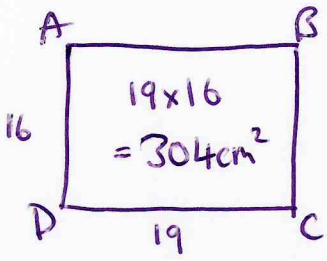
(Total for Question 7 is 3 marks)

8 Here is a diagram showing a rectangle, *ABCD*, and a circle.



BC is a diameter of the circle.

Calculate the percentage of the area of the rectangle that is shaded.
Give your answer correct to 1 decimal place.



$\frac{1}{2}(\pi \times 8^2) = 32\pi$

Shaded part $= 304 - 32\pi$

$= 203.469035\text{1cm}^2$

$\% = \frac{203.469}{304} \times 100 = 66.93060365\%$

66.9 %

(Total for Question 8 is 4 marks)

9 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.

linear A : B
10 : 9

Area 100 : 81

100:81

(2)

Square **E** has sides of length e cm.
Square **F** has sides of length f cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio $e : f$

Area E : F
144 : 100

linear 12 : 10

6:5

(2)

(Total for Question 9 is 4 marks)

10 The circumference of a circle is 10 m.

Work out the area of the circle.
Give your answer in terms of π.

$C = \pi \times d$

$10\text{m} = \pi \times d$

$\frac{10\text{m}}{\pi} = d$

$\frac{5\text{m}}{\pi} = r$

$\text{Area} = \pi \times r^2$

$= \pi \times \left(\frac{5}{\pi}\right)^2$

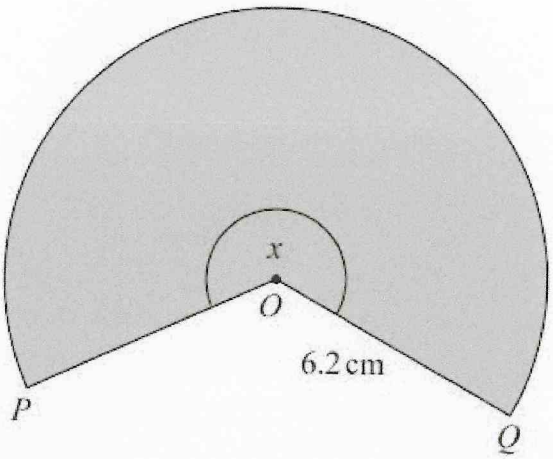
$= \pi \times \frac{25}{\pi^2}$

$= \frac{25\pi}{\pi^2}$

$\frac{25}{\pi}$ ..... m^2

(Total for Question 10 is 3 marks)

12 The diagram shows a shaded sector POQ of a circle with centre O and radius 6.2 cm.



The area of the shaded sector is $82.6\,cm^2$

Calculate the size of angle x.
Give your answer correct to 3 significant figures.

$$\text{Sector Area} = \frac{\theta}{360} \times \pi \times r^2$$

$$82.6 = \frac{x}{360} \times \pi \times 6.2^2$$

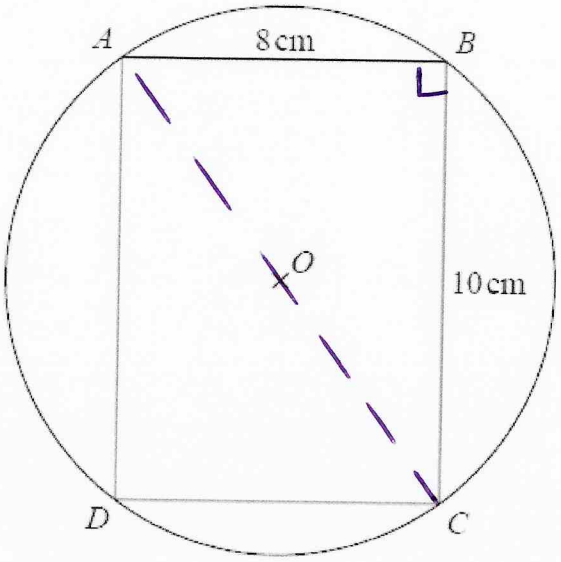
$$\frac{82.6 \times 360}{6.2^2 \pi} = x$$

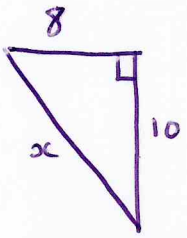
246 °

(Total for Question 12 is 2 marks)

$$246.234^\circ = x$$

12 The points A, B, C and D lie on a circle, centre O.
$ABCD$ is a rectangle.





$x = \sqrt{10^2 + 8^2}$

$= \sqrt{164}$

$= 2\sqrt{41}$

$AB = 8\,\text{cm}$ $BC = 10\,\text{cm}$

Work out the circumference of the circle.
Give your answer correct to 3 significant figures.

Diameter = $2\sqrt{41}$

Circumference = $\pi \times d$

$= \pi \times 2\sqrt{41}$

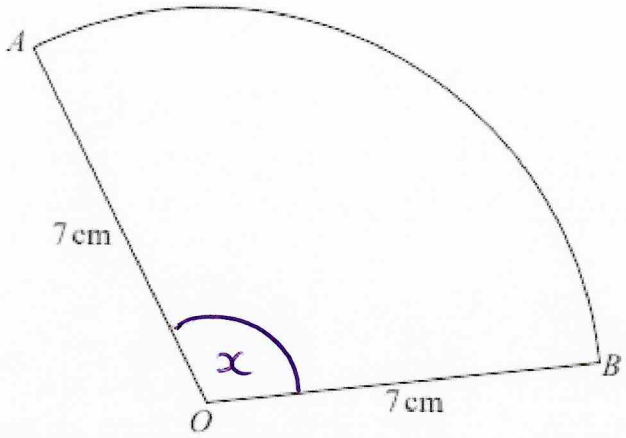
$= 40.232016\,13\,\text{cm}$

40.2

..... cm

(Total for Question 12 is 4 marks)

12 OAB is a sector of a circle with centre O and radius 7 cm.



The area of the sector is 40 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

Sector area $= \frac{x}{360} \times \pi \times r^2$

$40 = \frac{x}{360} \times \pi \times 7^2$

$\frac{40 \times 360}{49\pi} = x$

$93.5441^\circ = x$

Arc length $= \frac{93.5441}{360} \times \pi \times 14$

$= 11.428571 \text{ cm}$

$+ 7\text{cm}$

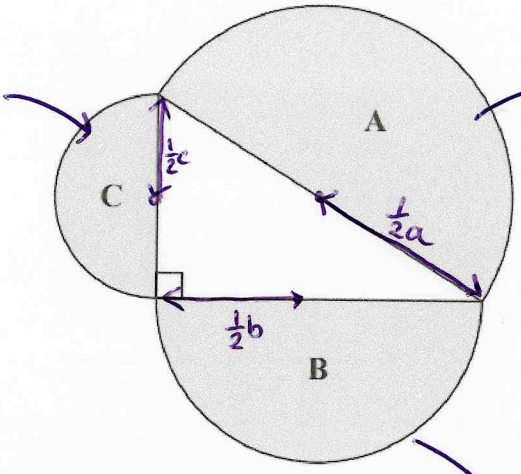
$+ 7\text{cm}$

$= 25.428571 \text{ cm}$

25.4 cm

(Total for Question 12 is 4 marks)

13 A right-angled triangle is formed by the diameters of three semicircular regions, **A**, **B** and **C** as shown in the diagram.



$\frac{1}{2} \times \pi \times \left(\frac{1}{2}c\right)^2$

$= \frac{1}{2}\pi \times \frac{1}{4}c^2$

$= \frac{1}{8}\pi c^2$

$\frac{1}{2} \times \pi \times \left(\frac{1}{2}a\right)^2$

$= \frac{1}{2}\pi \times \frac{1}{4}a^2$

$= \frac{1}{8}\pi a^2$

$\frac{1}{2} \times \pi \times \left(\frac{1}{2}b\right)^2$

$= \frac{1}{2}\pi \times \frac{1}{4}b^2$

$= \frac{1}{8}\pi b^2$

Show that

area of region **A** = area of region **B** + area of region **C**

$\div \pi$ | $\frac{1}{8}\pi a^2 = \frac{1}{8}\pi c^2 + \frac{1}{8}\pi b^2$ | $\div \pi$

$\frac{1}{8}a^2 = \frac{1}{8}c^2 + \frac{1}{8}b^2$

$\times 8$ | $a^2 = c^2 + b^2$ | $\times 8$

$\therefore$ Area A = Area C + Area B

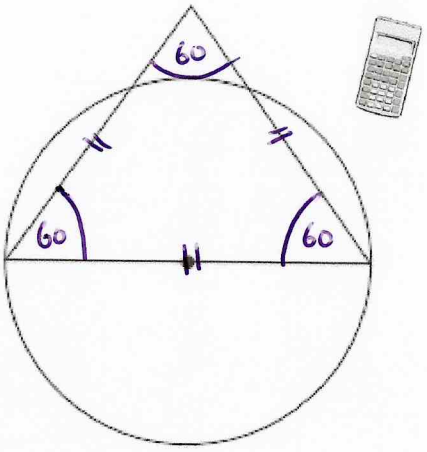
June 2022 – Paper 1H

(Total for Question 13 is 3 marks)

13 The diagram shows a circle and an equilateral triangle.

One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.

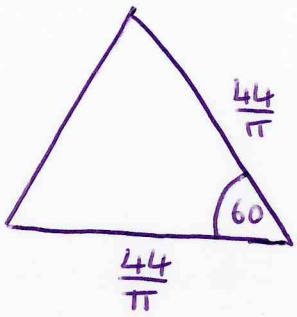
Work out the area of the triangle.
Give your answer correct to 3 significant figures.



$C = \pi \times d$

$44 = \pi \times d$

$\frac{44}{\pi} = \text{diameter}$



$\text{Area} = \frac{1}{2} ab \sin C$

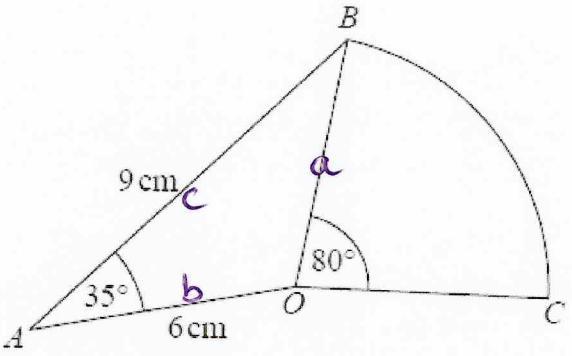
$= \frac{1}{2} \times \frac{44}{\pi} \times \frac{44}{\pi} \times \sin(60)$

$= 84.93882397 \text{cm}^2$

84.9 cm^2

(Total for Question 13 is 3 marks)

14 *OAB* is a triangle.
OBC is a sector of a circle, centre *O*.



Calculate the area of *OBC*.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos(35)$

$a^2 = 28.53157922\text{cm}$

$a = 5.341495972\text{cm}$

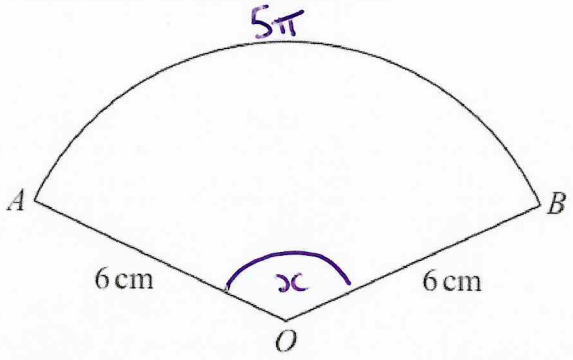
$\text{Area of OBC} = \frac{80}{360} \times \pi \times (5.34149)^2$

$= 19.91879993\text{cm}^2$

19.9 cm²

(Total for Question 14 is 4 marks)

15 OAB is a sector of a circle with centre O and radius 6 cm.



The length of the arc AB is 5π cm.

Work out, in terms of π, the area of the sector.
Give your answer in its simplest form.

$$\text{Arc} = \frac{x}{360} \times \pi \times d$$

$$5\pi = \frac{x}{360} \times \pi \times 12$$

$$\frac{5\pi \times 360}{12\pi} = x$$

$$\frac{1800\pi}{12\pi} = x$$

$$\frac{900\pi}{6\pi} = x$$

$$150^\circ = x$$

$$\text{Area} = \frac{150}{360} \times \pi \times 6^2$$

$$= \frac{5}{12} \times \pi \times 36$$

$$= \frac{180\pi}{12}$$

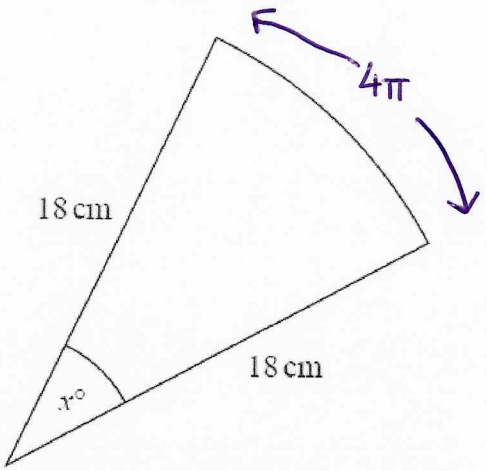
$$= \frac{90\pi}{6}$$

$$= 15\pi$$

15π ..... cm^2

(Total for Question 15 is 4 marks)

15 The diagram shows a sector of a circle of radius 18 cm.



The length of the arc is 4π cm.

Work out the value of x.

$$\text{Arc} = \frac{x}{360} \times \pi \times d$$

$$4\pi = \frac{x}{360} \times \pi \times 36$$

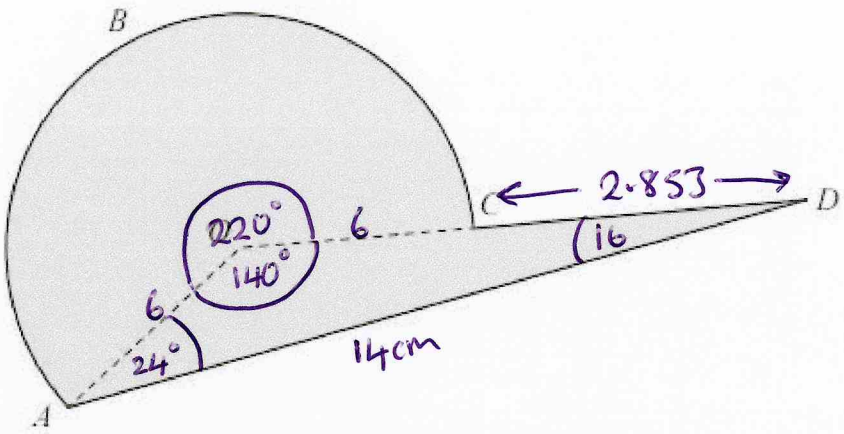
$$\frac{4\pi \times 360}{36\pi} = x$$

$$40 = x$$

$x =$ 40°

(Total for Question 15 is 3 marks)

16 Here is a shaded shape $ABCD$.

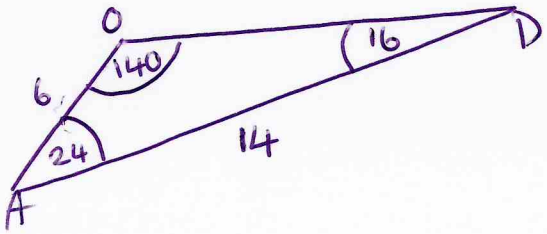


The shape is made from a triangle and a sector of a circle, centre O and radius 6 cm.
OCD is a straight line.

$AD = 14$ cm
Angle $AOD = 140°$
Angle $OAD = 24°$

Calculate the perimeter of the shape.
Give your answer correct to 3 significant figures.

Arc of ABC $= \frac{220}{360} \times \pi \times 12 = \frac{22}{3}\pi$



$OD \Rightarrow \frac{6}{\sin(16)} = \frac{OD}{\sin(24)}$

$\sin(24) \times \frac{6}{\sin(16)} = OD$

$OD = 8.853734107$

$CD = 2.85373$ cm

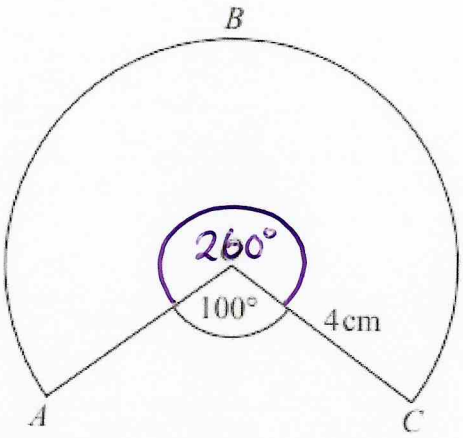
Perimeter $= \frac{22}{3}\pi + 14 + 2.8537$

$= 39.89208$ cm

39.9 cm

(Total for Question 16 is 5 marks)

16 The diagram shows a sector of a circle of radius 4 cm.



Work out the length of the arc ABC.
Give your answer correct to 3 significant figures.

$$\text{Arc} = \frac{260}{360} \times \pi \times 8$$

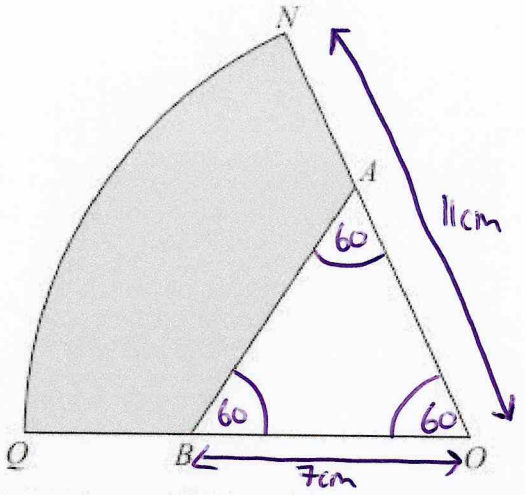
$$= \frac{52}{9}\pi$$

$$= 18.151422422 \text{ cm}$$

18.2 cm

(Total for Question 16 is 2 marks)

17



ONQ is a sector of a circle with centre *O* and radius 11 cm.

A is the point on *ON* and *B* is the point on *OQ* such that *AOB* is an equilateral triangle of side 7 cm.

Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.

Area of sector $\frac{60}{360} \times \pi \times 11^2 = \frac{121}{6}\pi$

Area of triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 7 \times 7 \times \sin 60 = \frac{49\sqrt{3}}{4}$

Shaded region $= \frac{121}{6}\pi - \frac{49\sqrt{3}}{4}$

$= 42.137829$

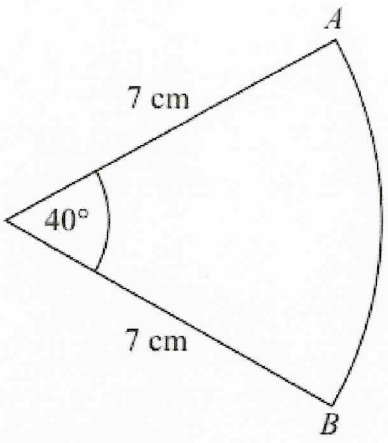
$\% = \frac{42.137829}{\frac{121\pi}{6}} \times 100$

$= 66.51018693$

66.5 %

(Total for Question 17 is 5 marks)

17 The diagram shows a sector of a circle of radius 7 cm.



Work out the length of arc AB.
Give your answer correct to 3 significant figures.

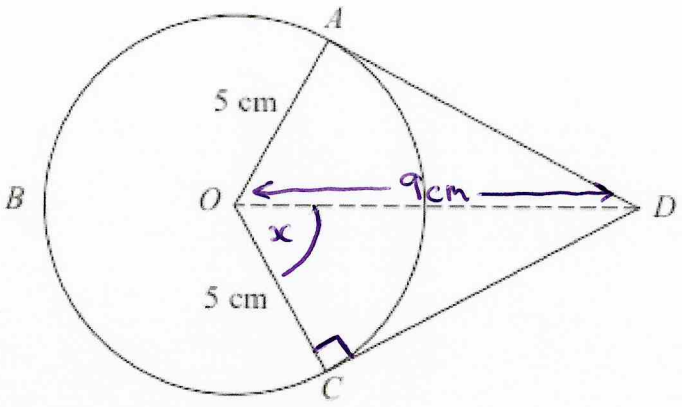
$$\text{Arc} = \frac{40}{360} \times \pi \times 14$$

$$= 4.886921906$$

4.89 cm

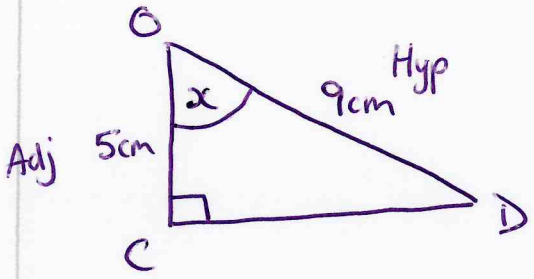
(Total for Question 17 is 2 marks)

18



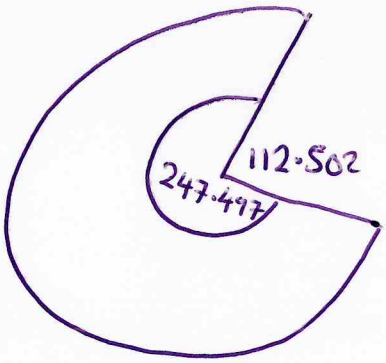
A, *B* and *C* are points on a circle of radius 5 cm, centre *O*.
DA and *DC* are tangents to the circle.
DO = 9 cm

Work out the length of arc *ABC*.
Give your answer correct to 3 significant figures.



$\cos^{-1}\left(\frac{5}{9}\right) = 56.2510114^\circ$

Angle COA = 112.502228°



$ABC = \frac{247.497}{360} \times \pi \times 10$

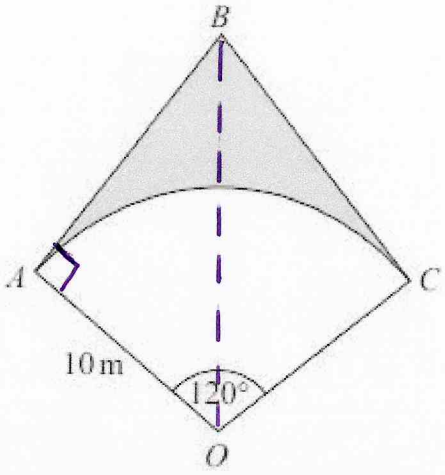
$= 21.59825506\text{cm}$

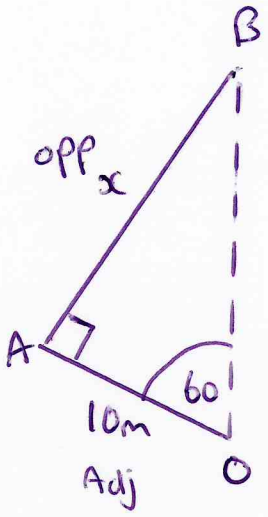
21.6 cm

June 2017 – Paper 3H

(Total for Question 18 is 5 marks)

20





OAC is a sector of a circle, centre O, radius 10 m.

BA is the tangent to the circle at point A.
BC is the tangent to the circle at point C.

Angle $AOC = 120°$

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

$\tan(60) \times 10 = x$

$10\sqrt{3} = x$

Area of BAO

$= \frac{b \times h}{2} = \frac{10 \times 10\sqrt{3}}{2} = 86.6025 cm^2$

Area of ABCO = $173.2050808 cm^2$

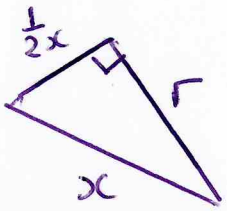
Area of sector ACO $= \frac{120}{360} \times \pi \times 10^2 = \frac{100}{3}\pi$

Area of shaded region

$173.2050808 - \frac{100}{3}\pi = 68.4853$ 68.5 m^2

(Total for Question 20 is 5 marks)

21 The diagram shows a circle, radius r cm and two regular hexagons.



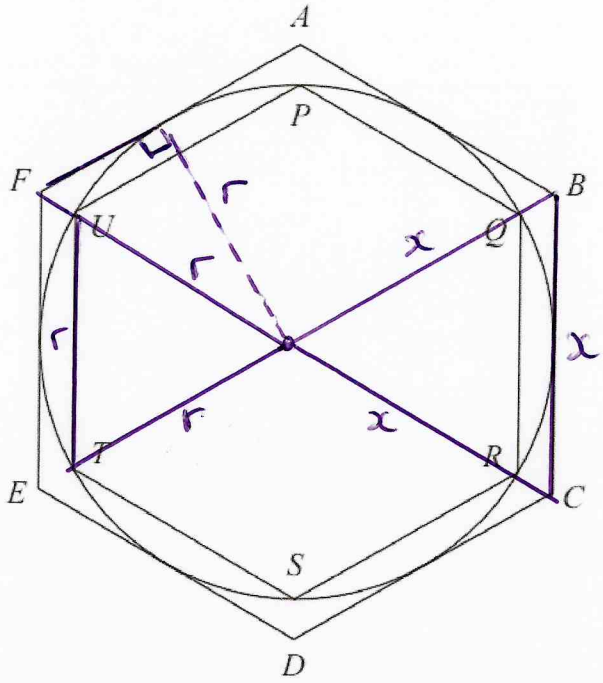
$r^2 + \left(\frac{1}{2}x\right)^2 = x^2$

$r^2 + \frac{1}{4}x^2 = x^2$

$r^2 = \frac{3}{4}x^2$

$\frac{4}{3}r^2 = x^2$

$\sqrt{\frac{4}{3}}r = x$



$x = \sqrt{\frac{4}{3}}r$

Each side of the larger hexagon $ABCDEF$ is a tangent to the circle.
Each side of the smaller hexagon $PQRSTU$ is a chord of the circle.

By considering perimeters, show that

$$3 < \pi < 2\sqrt{3}$$

Perimeter of small hexigon $= 6r$

Circumference of circle $= \pi \times 2r$

Perimeter of large hexigon $= 6 \times \sqrt{\frac{4}{3}}r$

$= 4\sqrt{3}r$

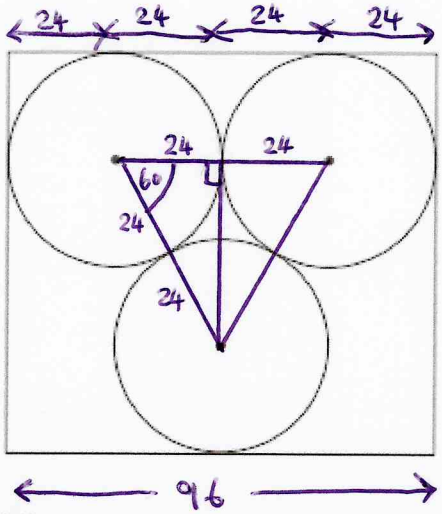
$6r < 2\pi r < 4\sqrt{3}r$

$6 < 2\pi < 4\sqrt{3}$

$3 < \pi < 2\sqrt{3}$

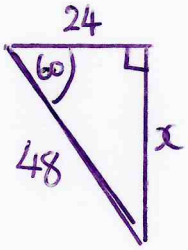
(Total for Question 21 is 4 marks)

21 The diagram shows 3 identical circles inside a rectangle.
Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.



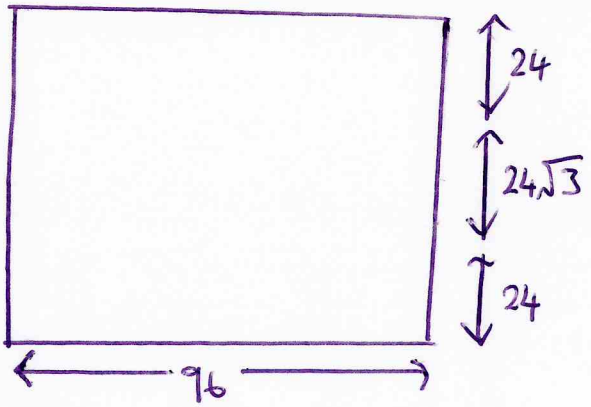
The radius of each circle is 24 mm.

Work out the area of the rectangle.
Give your answer correct to 3 significant figures.



$x = \sqrt{48^2 - 24^2}$

$= 24\sqrt{3}$



Area $= 96 \times (48 + 24\sqrt{3})$

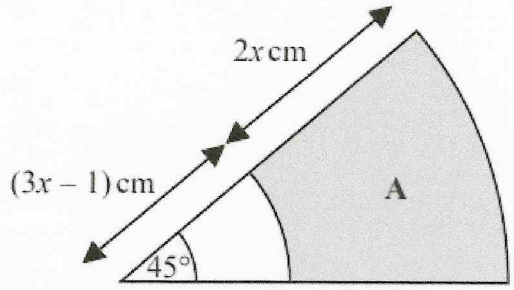
$= 8598.64506 \text{ mm}^2$

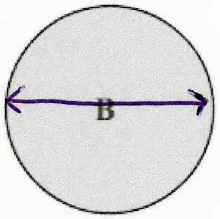
8600 mm²

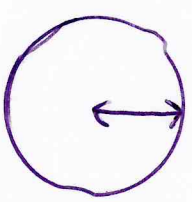
(Total for Question 21 is 4 marks)

22 The diagram shows two shaded shapes, **A** and **B**.

Shape **A** is formed by removing a sector of a circle with radius $(3x - 1)$ cm from a sector of the circle with radius $(5x - 1)$ cm.
Shape **B** is a circle of diameter $(2 - 2x)$ cm.







The area of shape **A** is equal to the area of shape **B**.

Find the value of x.
You must show all your working.

Area of shape A $= \left[\frac{45}{360} \times \pi \times (5x-1)^2\right] - \left[\frac{45}{360} \times \pi \times (3x-1)^2\right]$

$= \left[\frac{1}{8}\pi(25x^2 - 10x + 1)\right] - \left[\frac{1}{8}\pi(9x^2 - 6x + 1)\right]$

$= \left[\frac{1}{8}\pi(16x^2 - 4x)\right]$

Area of circle B $= \pi \times (1-x)^2 = \pi - 2x\pi + x^2\pi$

$\frac{1}{8}\pi(16x^2 - 4x) = \pi - 2x\pi + x^2\pi$ ÷ $\frac{1}{8}\pi$

$16x^2 - 4x = 8 - 16x + 8x^2$

$8x^2 + 12x - 8 = 0$ ÷4

$2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

$x = \frac{1}{2}$ or $x = -2$ ← x cannot be negative

$x = \frac{1}{2}$

(Total for Question 22 is 5 marks)